

Topics : Solution of Triangle, Vector, Sequence & Series, Continuity & Derivability

Type of Questions

M.M., Min.

Comprehension (no negative marking) Q.1 to Q.4	(3 marks, 3 min.)	[12, 12]
Single choice Objective (no negative marking) Q.5,6,7	(3 marks, 3 min.)	[9, 9]
Subjective Questions (no negative marking) Q.8,9,10	(4 marks, 5 min.)	[12, 15]

COMPREHENSION (FOR Q. 1 TO 4)

A regular heptagon (seven sides) is inscribed in a circle of radius 1. A_1, A_2, \dots, A_7 be its vertices, G_1 is centroid of $\triangle A_1 A_2 A_5$ and G_2 be centroid of $\triangle A_3 A_6 A_7$. P is centroid of $\triangle OG_1 G_2$, where O (origin) is centre of circumscribing circle.

1. Angle $\angle POA_1$ is equal to

- (A) $\frac{\pi}{7}$ (B) $\frac{2\pi}{7}$ (C) $\frac{5\pi}{7}$ (D) $\frac{6\pi}{7}$

2. OP is equal to

- (A) $\frac{10}{9}$ (B) $\frac{8}{9}$ (C) $\frac{1}{9}$ (D) 1

3. G_3 is such that centroid of triangle $G_1 G_2 G_3$ is O, then

- (A) $3OG_3 = OA_2$ (B) $3OG_2 = A_2 G_3$ (C) $2OG_3 = OA_2$ (D) $OG_3 = G_3 A_2$

4. PA_1 is equal to

- (A) $\frac{1}{9} \sqrt{82 - 18 \cos \frac{\pi}{7}}$ (B) $\frac{1}{9} \sqrt{82 + 18 \cos \frac{2\pi}{7}}$
(C) $\frac{1}{9} \sqrt{82 - 18 \sin \frac{2\pi}{7}}$ (D) None of these

5. If sine of the acute angle between two vectors $-3\hat{i} + 4\hat{j} + \hat{k}$ and $-2\hat{i} - \hat{j} - \hat{k}$ be $1 + \frac{1}{2}x - \frac{1}{8}x^2$ to ∞ then x is equal to

- (A) $\frac{155}{156}$ (B) $\frac{1}{156}$ (C) $-\frac{1}{156}$ (D) None of these

6. Let $\vec{a}, \vec{b}, \vec{c}$ be three unit vectors such that $|\vec{a} + \vec{b} + \vec{c}| = 1$ and $\vec{a} \perp \vec{b}$. If \vec{c} makes angles α, β with \vec{a}, \vec{b} respectively then $\cos\alpha + \cos\beta$ is equal to
 (A) $3/2$ (B) 1 (C) -1 (D) none of these
7. If $\vec{a} = (1, -1, 2)$, $\vec{b} = (-2, 3, 5)$, $\vec{c} = (2, -2, 4)$ and \hat{i} is the unit vector in positive x-direction, then $(\vec{a} - 2\vec{b} + 3\vec{c}) \cdot \hat{i}$ is equal to
 (A) 11 (B) 15 (C) 18 (D) 10
8. Find the sum of infinite terms of the series : $\frac{3}{2.4} + \frac{5}{2.4.6} + \frac{7}{2.4.6.8} + \frac{9}{2.4.6.8.10} + \dots$
9. Let $f : [a, b] \rightarrow \mathbb{R}$ a continuous positive function, differentiable on $[a, b]$. Prove that there exists $c \in (a, b)$ such that $\frac{f(b)}{f(a)} = e^{(b-a)\frac{f'(c)}{f(c)}}$
10. If \vec{a} and \vec{b} are two vectors such that $|\vec{a} + \vec{b}| = |\vec{a}|$ then prove that $2\vec{a} + \vec{b}$ is perpendicular to \vec{b} .

Answers Key

1. (A) 2. (C) 3. (A) 4. (A)
 5. (C) 6. (C) 7. (A) 8. $1/2$